Group-Blind Detection with Very Large Antenna Arrays in the Presence of Pilot Contamination

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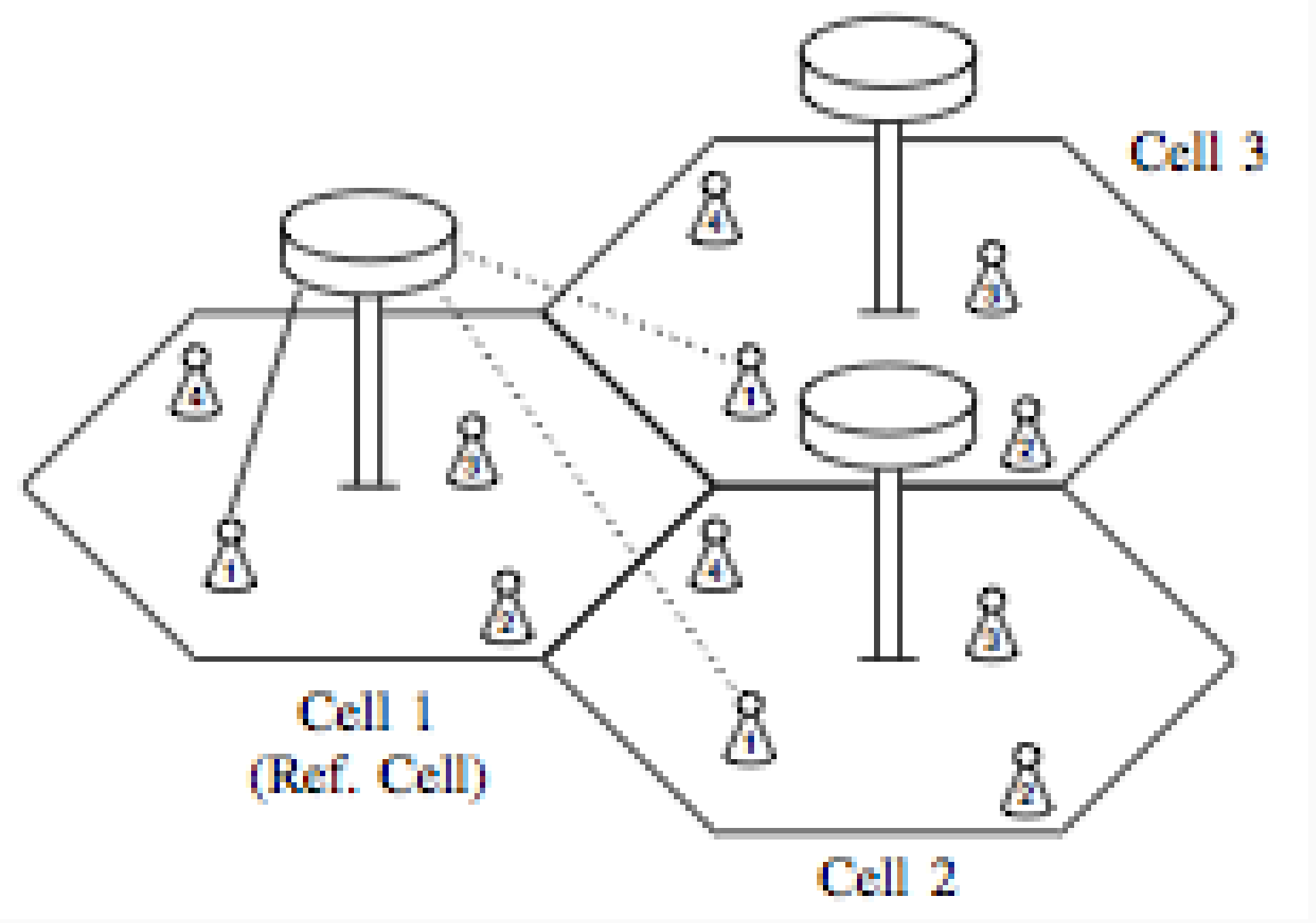
Introduction

Motivation



- Massive MIMO: key enabling technology to achieve 5G requirements
- Detector as simple as matched-filter is asymptotically optimal with perfect CSI
- Channel estimation based on pilots is standard practice in cellular networks
- Channel coherence time limits the maximum number of orthogonal pilots
- Pilots are reused in different cells: contamination arises

Problem statement: Pilot contamination limits the asymptotic rate achievable by massive MIMO. How to increase the asymptotic achievable rate while sticking to traditional channel estimation based on pilots reused in each cell?

System Model

System parameters and signals

Parameters

- n antennas at the BS
- L cells
- K single-antenna users per cell

Signals

$\boldsymbol{h}_{lk} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I})$, $\mathrm{Var}[x_{lk}] = P$, $\boldsymbol{n} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I})$

$$\boxed{\boldsymbol{y} = \sum_{l=1}^{L}\sum_{k=1}^{K} \boldsymbol{h}_{lk}\sqrt{\beta_{lk}}\, x_{lk} + \boldsymbol{n} = \sum_{l=1}^{L} \boldsymbol{G}_l \boldsymbol{x}_l + \boldsymbol{n}}$$

Channel estimation

Estimation of channel between reference BS (cell 1) and user k (within the cell)

$$\hat{\boldsymbol{g}}_{1k} = \left(\sum_{l\geq 1} \boldsymbol{g}_{lk} + \sqrt{\epsilon}\boldsymbol{\nu}_{1k}\right) \varphi_{1k}\, \beta_{1k}^{-1},$$

where $1/\epsilon$ is equal to the effective training SNR, $\boldsymbol{\nu}_{1k} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I})$, and

$$\varphi_{1k} = \frac{\beta_{1k}^2}{\epsilon + \sum_{l\geqslant 1} \beta_{lk}}.$$

In matrix form: channel estimations $\hat{\boldsymbol{G}}_l = [\hat{\boldsymbol{g}}_{l1}, \ldots, \hat{\boldsymbol{g}}_{lK}]$ and errors $\tilde{\boldsymbol{G}}_l = \boldsymbol{G}_l - \hat{\boldsymbol{G}}_l$.

Achievable rate

$$R_{1k} = \mathbb{E}[\log(1 + \gamma_{1k})]$$

where expectation is with respect to estimated channels, and SINR γ_{1k} is [3]

$$\gamma_{1k} = \frac{|\boldsymbol{w}_{1k}^\dagger \hat{\boldsymbol{g}}_{1k}|^2}{\mathbb{E}\left[\boldsymbol{w}_{1k}^\dagger \left(\frac{1}{P}\boldsymbol{I} + \tilde{\boldsymbol{g}}_{1k}\tilde{\boldsymbol{g}}_{1k}^\dagger + \sum_{j\neq k} \boldsymbol{g}_{1j}\boldsymbol{g}_{1j}^\dagger + \sum_{l>1}\sum_{j\geqslant 1} \boldsymbol{g}_{lj}\boldsymbol{g}_{lj}^\dagger \right) \boldsymbol{w}_{1k} \,\middle|\, \hat{\boldsymbol{G}}_1 \right]},$$

having denoted $\boldsymbol{w}_{1k}$ the linear receiver for user k. SINR γ_{1k} is equal to SINR of

$$\boldsymbol{y}' = \hat{\boldsymbol{G}}_1 \boldsymbol{x}_1 + \tilde{\boldsymbol{G}}_1 \tilde{\boldsymbol{x}}_1 + \sum_{l>1} \boldsymbol{G}_l \boldsymbol{x}_l + \boldsymbol{n},$$

where $\tilde{\boldsymbol{x}}_1$ is independent of $\boldsymbol{x}_1$ and has same covariance.

Proposed Group-Blind Detector

- Originally developed for CDMA [1], Group-Blind detection is adapted here to MIMO

$$\begin{aligned} \boldsymbol{w}_{1k} &= \dot{\boldsymbol{w}}_{1k} + \breve{\boldsymbol{w}}_{1k} \\ \dot{\boldsymbol{w}}_{1k} &\in \mathrm{range}\, \hat{\boldsymbol{G}}_1 \\ \breve{\boldsymbol{w}}_{1k} &\in \mathrm{range}\, \hat{\boldsymbol{G}}_1^\perp \cap \mathrm{range}\, [\boldsymbol{G}_1 \cdots \boldsymbol{G}_L] \end{aligned}$$

- $\dot{\boldsymbol{w}}_{1k}$ is derived on the basis of $\boldsymbol{y}_{\mathrm{in}} = \hat{\boldsymbol{G}}_1 \boldsymbol{x}_1 + \boldsymbol{n}$ according to MMSE

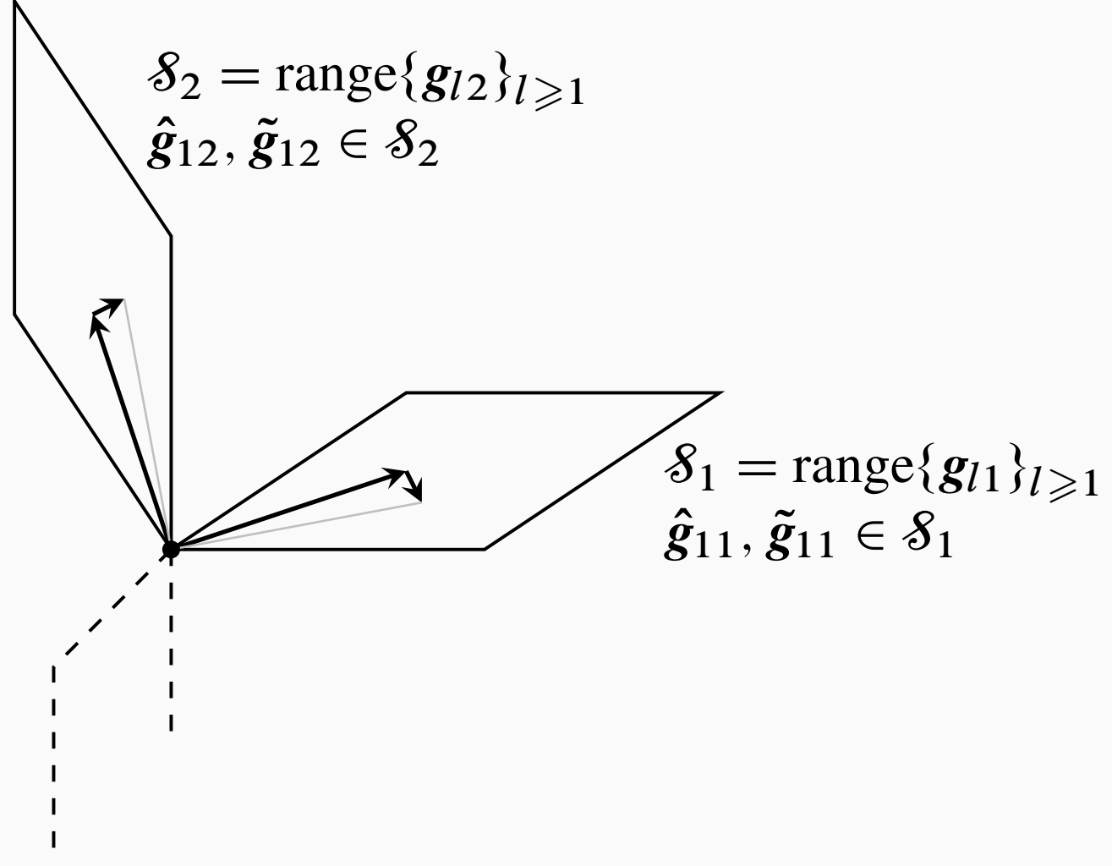
$$\dot{\boldsymbol{w}}_{1k} = \mathrm{argmin}_{\boldsymbol{w}}\, \mathbb{E}[\,|x_{1k} - \boldsymbol{w}^\dagger \boldsymbol{y}_{\mathrm{in}}|^2\,] = (\hat{\boldsymbol{G}}_1 \hat{\boldsymbol{G}}_1^\dagger + \tfrac{1}{P}\boldsymbol{I})^{-1} \hat{\boldsymbol{g}}_{1k}$$

- $\breve{\boldsymbol{w}}_{1k}$ is derived on the basis of the whole received signal according to MMSE [1]

$$\breve{\boldsymbol{w}}_{1k} = \mathrm{argmin}_{\boldsymbol{w}}\, \mathbb{E}[\,|x_{1k} - (\dot{\boldsymbol{w}}_{1k} + \boldsymbol{w})^\dagger \boldsymbol{y}|^2\,] = -\breve{\boldsymbol{U}}_{\hat{G}_1} \left(\breve{\boldsymbol{U}}_{\hat{G}_1}^\dagger \boldsymbol{C}_{\boldsymbol{y}'} \breve{\boldsymbol{U}}_{\hat{G}_1} \right)^{-1} \breve{\boldsymbol{U}}_{\hat{G}_1}^\dagger \boldsymbol{C}_{\boldsymbol{y}'} \dot{\boldsymbol{w}}_{1k}$$

where $\breve{\boldsymbol{U}}_{\hat{G}_1}$ spans range $\hat{\boldsymbol{G}}_1^\perp \cap$ range $[\boldsymbol{G}_1 \cdots \boldsymbol{G}_L]$ and $\boldsymbol{C}_{\boldsymbol{y}'}$ is the covariance of $\boldsymbol{y}'$.

Asymptotic Performance Analysis



Asymptotics (massive regime): $n \to \infty$, $K, L < \infty$.

Signal space properties in the massive regime [2,3]:

(i) $n^{-1} \boldsymbol{g}_{kl}^\dagger \boldsymbol{g}_{k'l'} \xrightarrow{\text{a.s.}} \beta_{kl}\, \delta_{kk'}\, \delta_{ll'}$, *i.e.*, channels are asymptotically almost surely orthogonal;

(ii) $\hat{\boldsymbol{g}}_{1k} \in \mathcal{S}_k = \mathrm{range}\{\boldsymbol{g}_{lk}\}_{l\geqslant 1}$ in high-SNR regime.

Results for $\boldsymbol{L = 2}$ (one dominant interfering cell)

Theorem

SINR γ_{1k} achieved by the proposed group-blind detector with $L = 2$ satisfies

$$\gamma_{1k} \xrightarrow{\text{a.s.}} \bar{\gamma}_{1k} = \left[1 + \frac{1}{(1 + \epsilon/\beta_{2k})^2}\right] \bar{\gamma}'_{1k}$$

where $\bar{\gamma}'_{1k} = \beta_{1k}^2 \beta_{2k}^{-2}$ is the SINR achieved with non-group-blind detection.

Define *asymptotic SINR gain*: $\bar{\eta}_{1k} = \bar{\gamma}_{1k} / \bar{\gamma}'_{1k}$.

Corollary. Asymptotic SINR γ_{1k} and gain $\bar{\eta}_{1k}$ with $L = 2$ satisfy:

$$\bar{\gamma}_{1k} \to 2\bar{\gamma}'_{1k}, \quad \bar{\eta}_{1k} \to 2, \qquad \text{as } \epsilon \to 0.$$

In brief: In the high-SNR regime, the asymptotic SINR achieved with group-blind detection is doubled compared to traditional detection.

Numerical Results

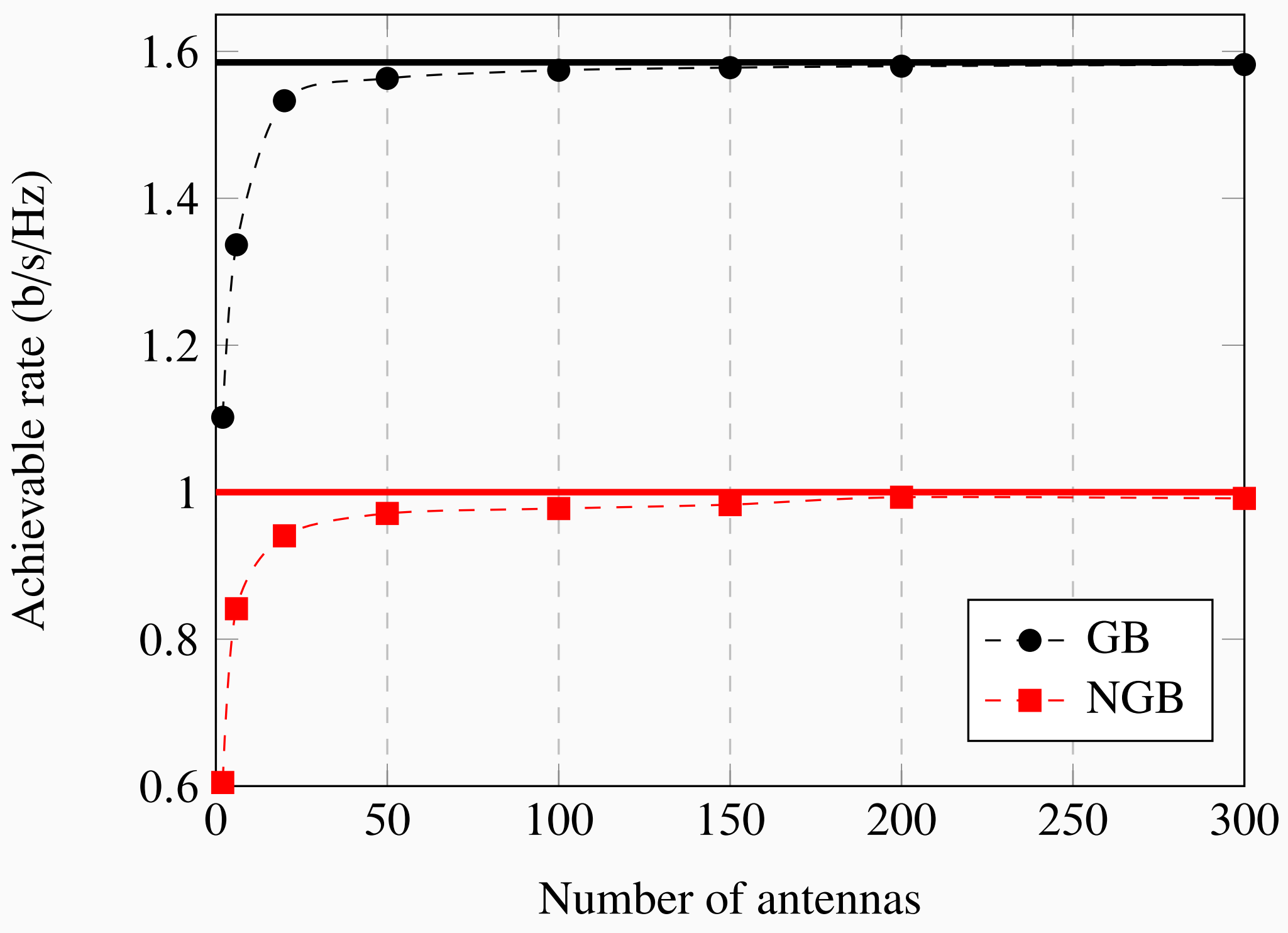


Fig. 1 Rate (b/s/Hz) vs. no. of antennas n with and without group-blind detection.
Scenario parameters: $\boldsymbol{L = 2}$, $K = 1$, SNR $= 20$ dB and $\beta_{11}/\beta_{21} = 0$ dB **(strong interference)**.

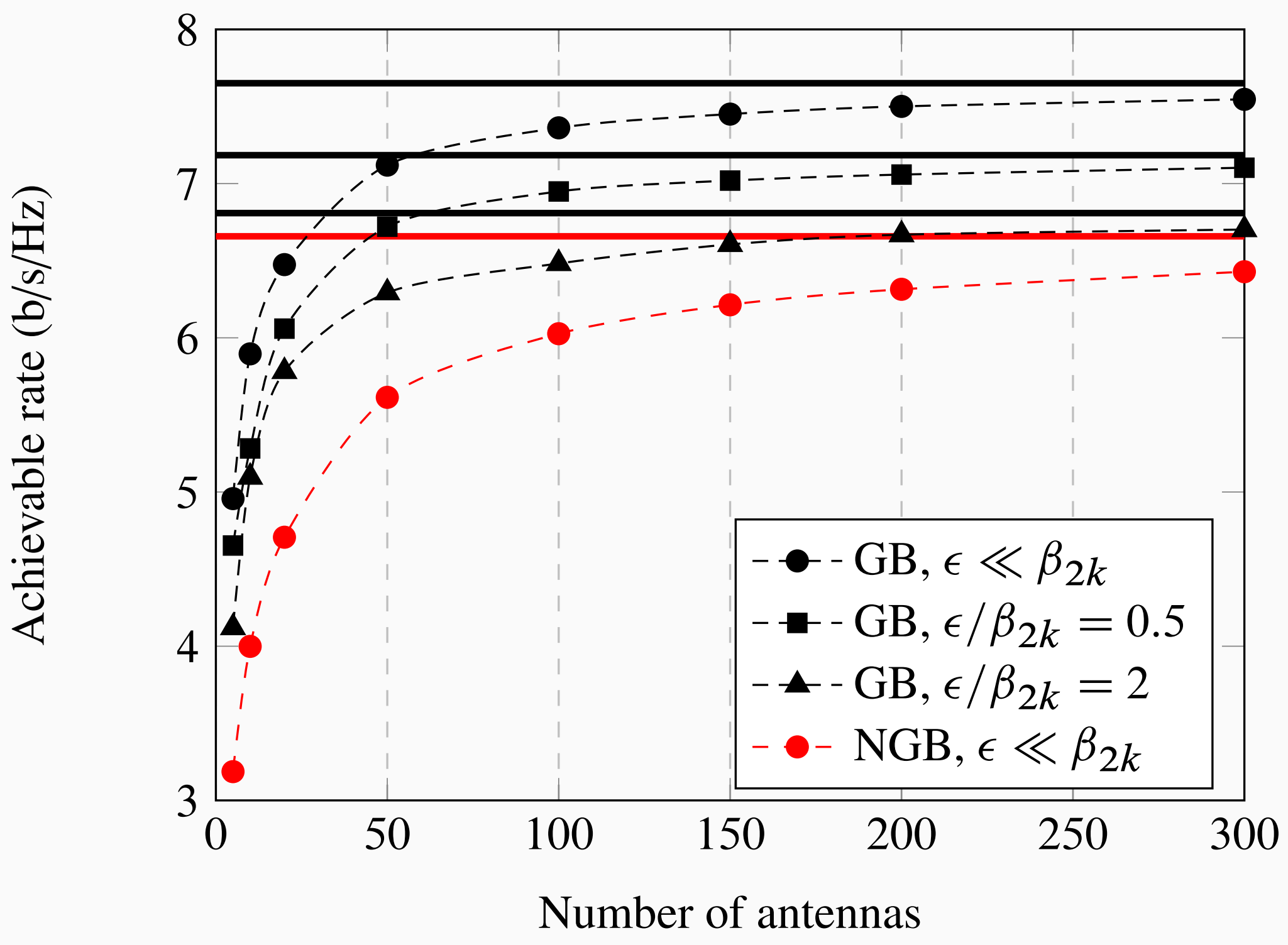


Fig. 2. Rate (b/s/Hz) vs. no. of antennas n with and without group-blind detection.
Scenario parameters: $\boldsymbol{L = 2}$, $K = 1$, SNR $= 10$ dB, and $\beta_{11}/\beta_{21} = 10$ dB **(weak interference)**.

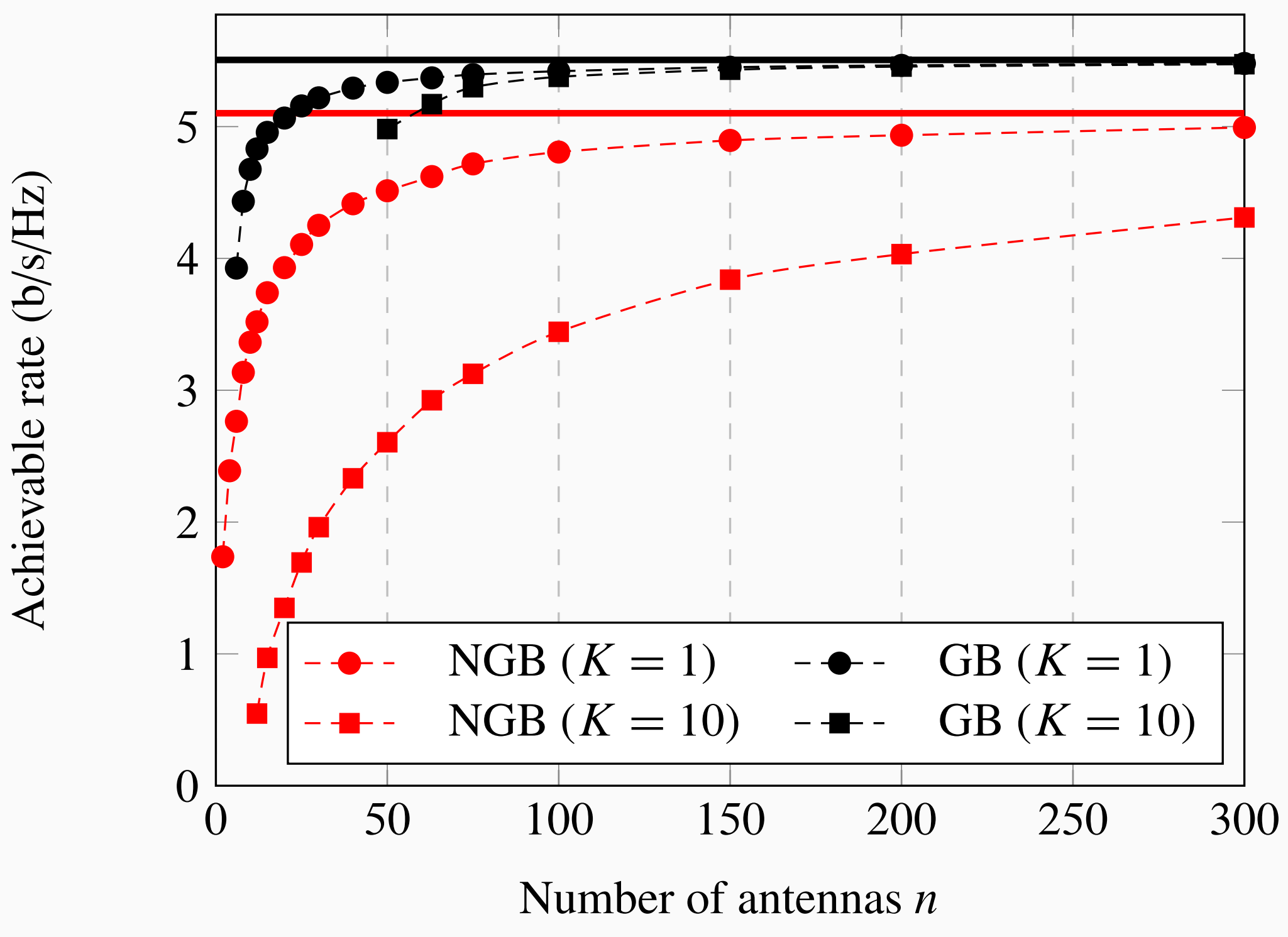


Fig. 3. Rate (b/s/Hz) vs. no. of antennas n with and without group-blind detection.
Scenario parameters: $\boldsymbol{L = 4}$, $\boldsymbol{K = 1}$ **or** $\boldsymbol{K = 10}$, SNR $= 10$ dB, $\beta_{1k}/\beta_{2k} = 10$ dB **(weak interference)**.

Asymptotic results in the general case and implementation in [4].

References

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[2] T. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590–3600, 2010.

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[4] G. C. Ferrante, G. Geraci, T. Q. S. Quek, and M. Z. Win, "Group-blind detection for uplink of massive MIMO systems," *IEEE Trans. on Signal Process.*, 2015, submitted for publication.