Capacity Bounds on Energy Harvesting Binary Symmetric Channels with Finite Battery

Zhengchuan Chen*†, Guido Carlo Ferrante*‡, Howard H. Yang*, and Tony Q. S. Quek*
*Information Systems Technology and Design Pillar, SUTD, Singapore 487372.
†College of Communication Engineering, Chongqing University, Chongqing 400044, China.
‡Laboratory for Information & Decision Systems (LIDS), MIT, MA 02139.
Emails: *{zhengchuan_chen@, guido_ferrante@, hao_yang@mymail., tonyquek@}sutd.edu.sg, †gcfer@mit.edu

***Abstract*—We investigate the capacity of energy harvesting binary symmetric channels with deterministic energy arrival process and finite battery size. Using an abstraction of the physical layer, binary symbols are transmitted. A cost function is associated with each transmitted symbol. Upper and lower bounds on the channel capacity are derived as functions of the normalized exponent of the cardinality of the set of feasible input sequences. Upper and lower bounds on the normalized exponent are established by studying supersets defined by relaxed constraints and employing a harvest-and-transmit signaling scheme, respectively. Numerical results validate that bounds on the exponent imply effective bounds on the channel capacity.**

I. Introduction

A promising feature of future wireless communication networks, in particular in view of the deployment of Internet-of-Things (IoT) networks [1], is the possibility of using the energy harvested from the environment to transmit information. The harvested energy can prolong the lifetime of the network and avoid frequent battery replacement. In typical sensor networks, most of the energy is consumed for information transmission [1]. Therefore, particular attention has been recently paid to searching for efficient transmission schemes and fundamental limits of energy harvesting communications [2], [3].

Energy harvesting communications systems are not memoryless due to the causality of energy harvesting and energy consumption. The memory of the system and the finite battery size have posed great challenges to the capacity analysis. An early result was that the capacity of the energy harvesting additive white Gaussian noise (AWGN) channel with infinite battery size is the same as the capacity of the AWGN channel with average power constraint [4]. This result extended the optimality of independent and identically distributed codebooks to energy harvesting channels with infinite battery by proving that the fraction of unfeasible codewords is vanishing in the large blocklength limit. However, this does not hold when the battery size is finite. A step toward understanding capacity in the finite battery regime was made in [5], [6] by assuming a noiseless channel and transforming the original energy harvesting channel to a timing channel. The problem is not trivial because of the memory in the sequence of transmitted symbols. In [7], [8], the capacity of the energy harvesting AWGN channel was characterized up to a constant gap of approximately 2.58 bits. Several contributions on the

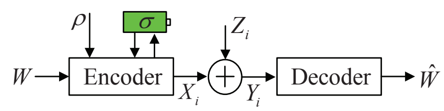

Fig. 1. Energy harvesting channel with (σ, ρ)-power constraint.

capacity analysis assumed that some sort of side information is available at receiver side [9], [10]. Capacity bounds have been established by regarding the energy level in the battery as channel state under different assumptions on the knowledge of the state at the transmitter [11], [12]. Recently, an energy harvesting system with deterministic energy arrival process and finite battery size was proposed for the study of energy harvesting AWGN channels [13]. In the proposed model, ρ units of energy are harvested in each time slot, and the battery can store at most σ units of energy. Thus, the constraint is referred to as (σ, ρ)-power constraint. In the original setup of [13], the channel is modelled as an AWGN channel and the transmitted symbol is a continuous variable.

In this work, we adopt the viewpoint of [13] and study the capacity of (σ, ρ)-power constrained energy harvesting channels by considering an abstraction of the physical layer. We model the channel as a binary symmetric channel (BSC) and consider binary transmitted symbols $x \in \{0, 1\}$. We derive the upper and lower bounds on the capacity of (σ, ρ)-power constrained energy harvesting binary symmetric channel (EHBSC) in terms of the normalized exponent of the cardinality of the set of feasible input sequences (hereinafter simply referred to as 'exponent'). By studying the constraints posed by the energy arrival process on the set of feasible inputs and employing specific signaling schemes based on a harvest-and-transmit strategy, we propose upper and lower bounds on the exponent and the channel capacity, respectively.

The rest of this paper is organized as follows. We introduce the EHBSC model in Section II. In Section III, we develop the capacity of special EHBSCs, and present bounds on the capacity of general cases as functions of the exponent of the cardinality of the feasible set. Section IV presents the upper and lower bounds on the exponent, which imply several bounds on capacity. Numerical results are provided in Section V and conclusion is drawn in Section VI.

II. System model

Consider the energy harvesting communication channel illustrated in Fig. 1. The transmitter is equipped with a battery that can store up to σ units of energy. Time is slotted and one binary symbol in $\mathbb{Z}_2 := \{\mathtt{0}, \mathtt{1}\}$ is transmitted within each time slot. Define a cost function $\mathsf{c}\colon \mathbb{Z}_2 \to [0, +\infty)$ that takes into account the energy spent for transmitting symbols. Without loss of generality, we set $\mathsf{c}(\mathtt{1}) = 1$, i.e., one unit of energy is used for transmitting symbol $\mathtt{1}$. We assume that $\mathsf{c}(\mathtt{0}) = 0$, i.e., symbol $\mathtt{0}$ is the zero-cost symbol. Hence, $\mathsf{c}(x) = x$. Given a sequence x^n, the cost of the sequence is the sum of the cost of individual symbols, which is equal to the Hamming weight of the sequence, i.e., the number of $\mathtt{1}$s in the sequence. It is assumed that $\rho \in [0, +\infty)$ units of energy arrives at the transmitter in each time slot, and that the transmitter stores energy if the battery is not full. Denote σ_i the amount of stored energy (state) at the beginning of time slot i. Such information is assumed to be known by the transmitter and unknown by the receiver. By assuming the battery is full at time $i = 0$, the state evolves as

$$\sigma_{i+1} = \min\{\sigma_i + \rho - \mathsf{c}(X_i), \sigma\}, \tag{1}$$

where $(X_1, X_2, \ldots)$ represents the transmitted symbols process. Note that because σ_{i+1} depends, in particular, on the symbol transmitted during slot i, $(\sigma_i)_{i=0}^{\infty}$ is a process with memory. A feasible input sequence $x^n := (x_1, \ldots, x_n)$ is a sequence of n symbols such that $\sigma_i \geqslant 0$ for $i = 1, \ldots, n$. The set of feasible input sequences is:

$$\mathcal{S}_n(\sigma, \rho) := \{x^n \in \mathbb{Z}_2^n : \sigma_i \geqslant 0,\ 0 \leqslant i \leqslant n\}. \tag{2}$$

We find it useful to define the following exponent:

$$v(\sigma, \rho) := \lim_{n\to\infty} \frac{1}{n} \log_2 |\mathcal{S}_n(\sigma, \rho)|, \tag{3}$$

where $|\mathcal{S}_n(\sigma, \rho)|$ represents the cardinality of $\mathcal{S}_n(\sigma, \rho)$. The existence of $v(\sigma, \rho)$ follows from the sub-additivity of $\log_2 |\mathcal{S}_n(\sigma, \rho)|$. The received symbol at slot i is

$$Y_i = X_i + Z_i \mod 2 \tag{4}$$

where $Z_i \sim \mathrm{Bern}(q)$ is a Bernoulli distributed noise with parameter $q \in [0, \frac{1}{2}]$. We assume that the channel is memoryless, i.e., $\{Z_i\}_{i\geqslant 1}$ is a sequence of i.i.d. random variables. We study the capacity of the EHBSC with (σ, ρ)-power constraint, denoted by $C(\sigma, \rho)$, in the following section.

III. Capacity Analysis

Capacity of the EHBSC is characterized in §III-A in special cases. General upper and lower bounds are presented in §III-B.

A. Special Cases

1) 'High' Energy Arrival Rate: If $\rho \geqslant 1$, then in each time slot the transmitter can select $\mathtt{1}$ or $\mathtt{0}$ freely irrespective of the battery size. Hence the energy harvesting channel is equivalent to the BSC with cross probability q, and $C(\sigma, \rho) = 1 - H_2(q)$ bits/use, where $H_2(q) := -q \log_2 q - (1-q)\log_2(1-q)$ is the binary entropy function.

2) 'Very Small' Battery Size: If $\rho < 1$ and $\sigma < 1 - \rho$, the transmitter cannot transmit symbol $\mathtt{1}$, irrespective of the time spent on harvesting energy. Thus, $C(\sigma, \rho) = 0$ for $\sigma + \rho < 1$.

3) Infinite Battery Size: If the battery size is infinite, i.e., $\sigma = \infty$, it can store all the unused harvested energy for future transmissions. Following the save-and-transmit strategy [4], capacity of the EHBSC is given by:

$$C(\infty, \rho) = \begin{cases} H_2(\rho * q) - H_2(q), & \text{if } \rho < \frac{1}{2}, \\ 1 - H_2(q), & \text{otherwise} \end{cases}, \tag{5}$$

where $\rho * q := \rho(1-q) + q(1-\rho)$.

B. General Cases

We focus on the EHBSC with (σ, ρ)-power constraint satisfying $\rho \in (0, 1)$ and $\sigma \geqslant 1 - \rho$, referred to as *general* EHBSC. We can bound the capacity of (σ, ρ)-power constrained general EHBSC as follows.

Theorem 1. *The capacity of (σ, ρ)-power constrained EHBSC with crossover probability q satisfies*

$$H_2\big(H_2^{-1}(v(\sigma, \rho)) * q\big) - H_2(q) \leqslant C(\sigma, \rho) \leqslant \min\{C(\infty, \rho), v(\sigma, \rho)\}. \tag{6}$$

Proof. Let $\mathcal{F}_n$ represents the set of all probability distributions supported almost surely on $\mathcal{S}_n(\sigma, \rho)$. Then, the equivalence between the operational definition of capacity and its information-theoretical counterpart is given by [13]

$$C(\sigma, \rho) = \lim_{n\to\infty} \frac{1}{n} \sup_{P_{X^n} \in \mathcal{F}_n} I(X^n; Y^n). \tag{7}$$

From the binary entropy-power inequality [14], it results

$$H_2^{-1}\left(\lim_{n\to\infty} \frac{1}{n} H(Y^n)\right) \geqslant H_2^{-1}\left(\lim_{n\to\infty} \frac{1}{n} H(X^n)\right) * q.$$

Hence, one has

$$\frac{1}{n} I(X^n; Y^n) = \frac{1}{n} H(Y^n) - H_2(q) \geqslant H_2\left(H_2^{-1}\left(\frac{1}{n} H(X^n)\right) * q\right) - H_2(q). \tag{8}$$

Since both $H_2(p * q)$ and $H_2^{-1}(p)$ increase as a function of p, the following inequality holds:

$$\sup_{P_{X^n} \in \mathcal{F}_n} \frac{I(X^n; Y^n)}{n} \geqslant H_2\left(H_2^{-1}\left(\sup_{P_{X^n} \in \mathcal{F}_n} \frac{H(X^n)}{n}\right) * q\right) - H_2(q) \stackrel{\text{(a)}}{=} H_2\left(H_2^{-1}\left(\frac{\log_2 |\mathcal{S}_n(\sigma, \rho)|}{n}\right) * q\right) - H_2(q), \tag{9}$$

where (a) follows from the uniform distribution over $\mathcal{S}_n(\sigma, \rho)$ being optimum. Taking the limit as $n \to \infty$ yields the lower bound in (6) by continuity of $H_2(\cdot)$ (cf. (3)).

To derive the upper bound, observe that

$$\lim_{n\to\infty} \sup_{P_{X^n} \in \mathcal{F}_n} \frac{1}{n} I(X^n; Y^n) \leqslant \lim_{n\to\infty} \sup_{P_{X^n} \in \mathcal{F}_n} \frac{1}{n} H(X^n) = \lim_{n\to\infty} \frac{\log_2 |\mathcal{S}_n(\sigma, \rho)|}{n} = v(\sigma, \rho). \tag{10}$$

Noting that $C(\sigma,\rho)$ is naturally bounded by $C(\infty,\rho)$ and combining (7), (9), and (10) yield the upper bound in (6). $\square$

IV. Upper and Lower Bounds on $v(\sigma,\rho)$

In this section, we present upper and lower bounds on $v(\sigma,\rho)$, which imply upper and lower bounds on the channel capacity $C(\sigma,\rho)$, respectively, by virtue of Theorem 1.

A. Upper Bounds on $v(\sigma,\rho)$

By telescoping the minimum operation in (1), one has

$$\sigma_i = \min\left(\sigma,\, \sigma+\rho-x_i,\, \ldots,\, \sigma+i\rho-\sum_{j=1}^{i}\mathsf{c}(x_j)\right) \geqslant 0.$$

Therefore, one can equivalently express (2) as follows [13]:

$$\mathcal{S}_n(\sigma,\rho) = \Bigg\{x^n \in \mathbb{Z}_2^n : \sum_{i=j+1}^{j+l}\mathsf{c}(x_i) \leqslant \sigma + l\rho, \\ 0 \leqslant j \leqslant n-l,\ 1 \leqslant l \leqslant n\Bigg\}. \quad (11)$$

The above (σ,ρ)-power constraint indicates that an input sequence is feasible if and only if all subsequences do not violate the energy constraint, i.e., the Hamming weight of each subsequence is not larger than the battery size plus the amount of energy harvested during its transmission.

To derive an upper bound on $v(\sigma,\rho)$, let us remove some of the constraints that define the set $\mathcal{S}_n(\sigma,\rho)$. In particular, for fixed σ and ρ, define the set

$$\mathcal{S}_n^{(l)} := \left\{x^n \in \mathbb{Z}_2^n : \sum_{i=j+1}^{j+l} x_i \leqslant \sigma + l\rho,\ 0 \leqslant j \leqslant n-l\right\}. \quad (12)$$

Then, $\mathcal{S}_n(\sigma,\rho) = \bigcap_{l=1}^{n}\mathcal{S}_n^{(l)}$, and in particular:

$$|\mathcal{S}_n(\sigma,\rho)| \leqslant |\mathcal{S}_n^{(l)}|, \quad l = 1,\ldots,n. \quad (13)$$

To study $|\mathcal{S}_n^{(l)}|$, let us consider the following partition of $\mathcal{S}_n^{(l)}$ in K subsets $\{\mathcal{S}_{n1}^{(l)},\ldots,\mathcal{S}_{nK}^{(l)}\}$, where $\mathcal{S}_{nk}^{(l)}$ is the subset of sequences in $\mathcal{S}_n^{(l)}$ with the l trailing (rightmost) symbols representing $k-1$ in binary notation, i.e.:

$$\mathcal{S}_{nk}^{(l)} := \left\{x^n \in \mathcal{S}_n^{(l)} : (x_{n-l+1}^n)_{10} = k-1\right\}, \quad (14)$$

where $(\cdot)_{10}$ denotes the operator that outputs the decimal value of the binary string in the argument (e.g., $(\mathtt{110})_{10} = 6$). In general, there can be $K := 2^l$ possible subsets, some of which can be empty because of the Hamming constraint given in (12). Let $z_{nk}^{(l)} := |\mathcal{S}_{nk}^{(l)}|$ be the cardinality of subset k. We group these cardinalities in a vector $\boldsymbol{z}_n^{(l)} := (z_{n1}^{(l)},\ldots,z_{nK}^{(l)})^\mathsf{T}$. The following relation between $\boldsymbol{z}_n^{(l)}$ and $\boldsymbol{z}_{n+1}^{(l)}$ holds:

$$\boldsymbol{z}_{n+1}^{(l)} = \boldsymbol{A}_l\boldsymbol{z}_n^{(l)}, \quad (15)$$

where $(\boldsymbol{A}_l)_{ij} = 1$ if each sequence $x^n \in \mathcal{S}_{ni}^{(l)}$ can generate a sequence $x^{n+1} \in \mathcal{S}_{(n+1)j}^{(l)}$ by padding $\mathtt{0}$ or $\mathtt{1}$ at position $n+1$, and $(\boldsymbol{A}_l)_{ij} = 0$ otherwise. Note that $\boldsymbol{A}_l \in \mathbb{R}^{K\times K}$ depends on l but it does not depend on n.

Based on the spectral properties of $\boldsymbol{A}_l$, we have the following upper bound on $v(\sigma,\rho)$.

Theorem 2. *The exponent $v(\sigma,\rho)$ satisfies*

$$v(\sigma,\rho) \leqslant \min_{l\geqslant 1}\log_2\lambda_{\max}(\boldsymbol{A}_l), \quad (16)$$

where $\lambda_{\max}(\boldsymbol{A}_l)$ is the maximum eigenvalue of $\boldsymbol{A}_l$.

Proof. By iteratively applying (15) backward we have

$$\boldsymbol{z}_n^{(l)} = \boldsymbol{A}_l^{n-l}\boldsymbol{z}_l^{(l)} = \boldsymbol{A}_l^{n-l}\boldsymbol{1}_K,$$

where $\boldsymbol{1}_K$ is a length-K vector of 1s. Note that $\boldsymbol{z}_l^{(l)} = \boldsymbol{1}_K$ since there is just one subsequence in each subset $\mathcal{S}_{lk}^{(l)}$ (cf. (14)). By using (15) again, we have

$$|\mathcal{S}_n^{(l)}| = \sum_{k=1}^{K} z_{nk}^{(l)} = \|\boldsymbol{z}_n^{(l)}\|_1.$$

Therefore, we can express the cardinality of $\mathcal{S}_n^{(l)}$ in terms of $\boldsymbol{A}_l$ only:

$$|\mathcal{S}_n^{(l)}| = \|\boldsymbol{A}_l^{n-l}\boldsymbol{1}_K\|_1. \quad (17)$$

Now we bound the ℓ_1-norm as follows:

$$\|\boldsymbol{A}_l^{n-l}\boldsymbol{1}_K\|_1 \leqslant \|\boldsymbol{A}_l^{n-l}\|_1\|\boldsymbol{1}_K\|_1 = \|\boldsymbol{A}_l^{n-l}\|_1 K,$$

where the matrix norm is induced by the ℓ_1-norm and the inequality follows from submultiplicativity of the norm. Using Gel'fand's formula yields

$$\lim_{n\to\infty}\|\boldsymbol{A}_l^{n-l}\boldsymbol{1}_K\|_1^{\frac{1}{n-l}} \leqslant \lim_{n\to\infty}\|\boldsymbol{A}_l^{n-l}\|_1^{\frac{1}{n-l}}K^{\frac{1}{n-l}} = \lambda_{\max}(\boldsymbol{A}_l), \quad (18)$$

being l fixed. From (17) and (18) we can upper bound the exponent of $|\mathcal{S}_n(\sigma,\rho)|$ as follows:

$$\begin{aligned} v(\sigma,\rho) &\leqslant \lim_{n\to\infty}\frac{1}{n-l}\log_2|\mathcal{S}_n^{(l)}| \\ &= \lim_{n\to\infty}\log_2\|\boldsymbol{A}_l^{n-l}\boldsymbol{1}_K\|_1^{\frac{1}{n-l}} \leqslant \log_2\lambda_{\max}(\boldsymbol{A}_l). \end{aligned}$$

Tightening the bound with respect to l yields the theorem. $\square$

In general, $\lambda_{\max}(\boldsymbol{A}_l)$ can be evaluated numerically. For some specific l, we can find the structure of $\boldsymbol{A}_l$ and derive $\lambda_{\max}(\boldsymbol{A}_l)$ explicitly. In particular, consider l satisfying $l-1 \leqslant \sigma + l\rho < l$. Then, $l \in \left(\frac{\sigma}{1-\rho}, \frac{\sigma+1}{1-\rho}\right]$. As $\frac{1}{1-\rho} > 1$ holds, such integer l does always exist. The smallest integer is $l_1 := \left\lfloor\frac{\sigma}{1-\rho}\right\rfloor + 1$. When $l = l_1$, the structure of $\boldsymbol{A}_{l_1}$ is

$$\boldsymbol{A}_{l_1} := \left[\begin{array}{c|c} \boldsymbol{B}_{l_1} & \boldsymbol{B}_{l_1} \\ \hline \begin{matrix} & 1 \\ & 0 \end{matrix} & \begin{matrix} & 0 \\ & 0 \end{matrix} \end{array}\right], \ \boldsymbol{B}_{l_1} := \begin{bmatrix} \boldsymbol{1}_2 & & \\ & \ddots & \\ & & \boldsymbol{1}_2 \end{bmatrix},$$

where the size of $\boldsymbol{B}_{l_1}$ is $(2^{l_1}-2)\times 2^{l_1-1}$. It can be verified that the characteristic polynomial of $\boldsymbol{A}_{l_1}$ is

$$F(\lambda) = \lambda^{2^{l_1}-l_1}\left(\lambda^{l_1} - \sum_{i=0}^{l_1-1}\lambda^i\right) =: \lambda^{2^{l_1}-l_1}\bar{F}(\lambda). \quad (19)$$

The largest eigenvalue of $\boldsymbol{A}_{l_1}$, $\lambda_{\max}(\boldsymbol{A}_{l_1})$, is the largest real

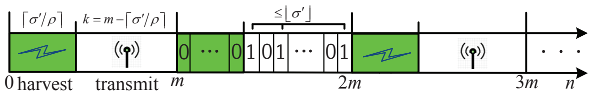

Fig. 2. The harvest-and-transmit scheme. The transmitter alternates harvest and transmission phases of length $m-k$ and k symbols, respectively.

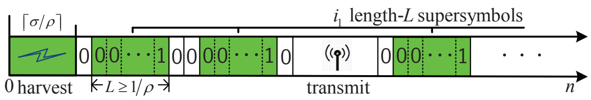

Fig. 3. A modified harvest-and-transmit scheme which employs length-L supersymbol $\texttt{00}\cdots\texttt{01}$ satisfying $L\rho \geqslant 1$.

root of $F(\lambda)=0$, i.e., the largest real root of $\bar{F}(\lambda)$.

B. Lower Bounds on $v(\sigma,\rho)$

To lower bound $v(\sigma,\rho)$, we bound the cardinality of the feasible set by employing a specific transmission strategy referred to as harvest-and-transmit strategy [4]. See Fig. 2. According to the harvest-and-transmit strategy, transmitter alternates harvest and transmission phases of length $m-k$ and k symbols, respectively. The battery can store energy up to a preset level σ' during the harvest phase, and can use the stored energy to send k symbols in the transmission phase. The following lemma shows a lower bound on $v(\sigma,\rho)$.

Lemma 1. *The exponent of $|\mathcal{S}_n(\sigma,\rho)|$ satisfies*

$$v(\sigma,\rho) \geqslant \max_{\substack{1-\rho\leqslant\sigma'\leqslant\sigma\\ k>0}} \frac{1}{k+\lceil\sigma'/\rho\rceil}\log_2\left[\sum_{i=0}^{\lfloor\sigma'\rfloor}\binom{k}{i}\right]. \tag{20}$$

Proof. Suppose the length-n input sequence is divided into subsequences of length m, each subsequence representing a harvest-and-transmit cycle. The harvest phase lasts for $\lceil\sigma'/\rho\rceil$ symbols, therefore the transmission phase length is $k := m - \lceil\sigma'/\rho\rceil$. During the transmission phase, at most $\lfloor\sigma'\rfloor$ symbols $\texttt{1}$ can be transmitted. Accordingly, the number of sequences that we can transmit is $\sum_{i=0}^{\lfloor\sigma'\rfloor}\binom{k}{i}$. Then, $|\mathcal{S}_n(\sigma,\rho)|$ is lower bounded as follows, $|\mathcal{S}_n(\sigma,\rho)| \geqslant \left(\sum_{i=0}^{\lfloor\sigma'\rfloor}\binom{k}{i}\right)^{n/m}$, hence $v(\sigma,\rho)$ is lower bounded by $(k+\lceil\sigma'/\rho\rceil)^{-1}\log_2\left(\sum_{i=0}^{\lfloor\sigma'\rfloor}\binom{k}{i}\right)$. The statement follows by tightening the bound with respect to $\sigma'\in[1-\rho,\sigma]$ and $k>0$. ☐

Note that the cost of symbols $\texttt{1}$ can be reduced from 1 to $1-\rho$ by using the energy harvested during the slot. This improvement allows to lower bound $v(\sigma,\rho)$ as follows.

Theorem 3. *The exponent of $|\mathcal{S}_n(\sigma,\rho)|$ satisfies*

$$v(\sigma,\rho) \geqslant \max_{\substack{1-\rho\leqslant\sigma'\leqslant\sigma\\ k>0}} \frac{1}{k+\lceil\sigma'/\rho\rceil}\log_2\left[\sum_{i=0}^{\lfloor\sigma'/(1-\rho)\rfloor}\binom{k}{i}\right]. \tag{21}$$

Proof. By harvesting energy in the slot where a symbol $\texttt{1}$ is transmitted, the actual cost of transmission is $1-\rho$. Therefore, the number of $\texttt{1}$s that can be transmitted in the transmission phase is at most $\lfloor\sigma'/(1-\rho)\rfloor$. ☐

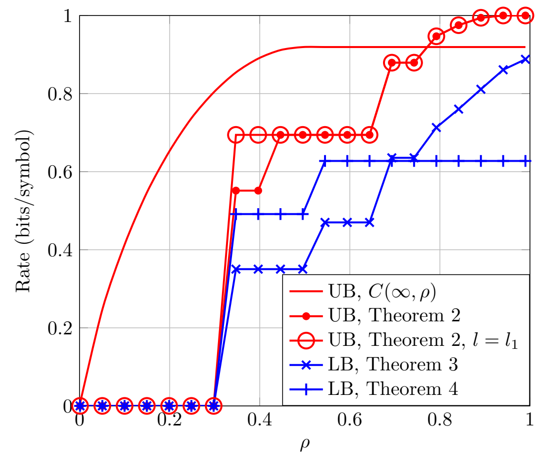

Fig. 4. Bounds on $C(\sigma,\rho)$ as a function of ρ for $\sigma=0.7$ and $q=0.01$.

By slightly modifying the harvest-and-transmit scheme, we can constrain the transmitter to use either $\texttt{0}$ or a length-L unit-weight subsequence $\texttt{00}\cdots\texttt{01}$ during which the transmitter always harvests energy (cf. Fig. 3). Accordingly, we can think of the length-L subsequence as a 'supersymbol'. The peculiarity of this scheme is that, by choosing a sufficiently large L satisfying $L\rho\geqslant 1$, it allows to use infinitely many times the supersymbol regardless of the battery level (provided that $\sigma\geqslant 1-\rho$). This implies the following lower bound on $v(\sigma,\rho)$.

Theorem 4. *Given $\sigma\geqslant 1-\rho$. For $L\geqslant 1/\rho$, the exponent of $|\mathcal{S}_n(\sigma,\rho)|$ satisfies*

$$v(\sigma,\rho) \geqslant \max_{0<\alpha\leqslant 1-1/L}(1-\alpha)H_2\left(\frac{\alpha}{1-\alpha}\cdot\frac{1}{L-1}\right). \tag{22}$$

Proof. Let i_0 and i_1 be the number of $\texttt{0}$s and length-L supersymbols transmitted, respectively. Transmission starts with a number of $\texttt{0}$s equal to $\lceil\sigma/\rho\rceil$; then i_0 symbols $\texttt{0}$ and i_1 supersymbols can be transmitted in any of the possible combinations, which are $\binom{i_0+i_1}{i_1}$. The necessary number of time slots is $k := i_0 + Li_1$ (cf. Fig. 3), hence the following exponent is achievable:

$$\begin{aligned} v(\sigma,\rho) &\overset{(a)}{\geqslant} \lim_{k\to\infty}\frac{1}{k+\lceil\sigma/\rho\rceil}\log_2\binom{k-i_1(L-1)}{i_1} \\ &\overset{(b)}{\geqslant} \lim_{k\to\infty}\left[\frac{k-i_1(L-1)}{k+\lceil\sigma/\rho\rceil}H_2\left(\frac{i_1}{k-i_1(L-1)}\right)\right. \\ &\qquad\left. - \frac{\log_2[k+1-i_1(L-1)]}{k+\lceil\sigma/\rho\rceil}\right] \end{aligned}$$

where: (a) follows from $n = k+\lceil\sigma/\rho\rceil$ and by counting the number of possible sequences that can be transmitted; and (b) follows from a standard lower bound of the binomial coefficient. The statement follows by setting $\alpha = i_1(L-1)/k$ and by letting $k\to\infty$, and by maximizing over $0<\alpha<1-1/L$. ☐

V. Numerical Results

In this section, we present numerical results and evaluate the bounds established in the previous section. Figs. 4 and 5 show upper and lower bounds on capacity as a function of ρ

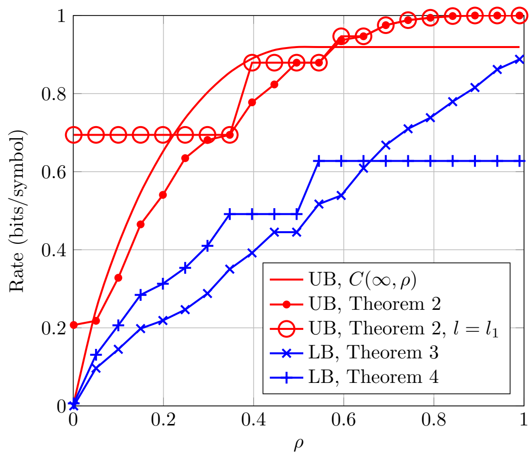

Fig. 5. Bounds on $C(\sigma, \rho)$ as a function of ρ for $\sigma = 1.3$ and $q = 0.01$.

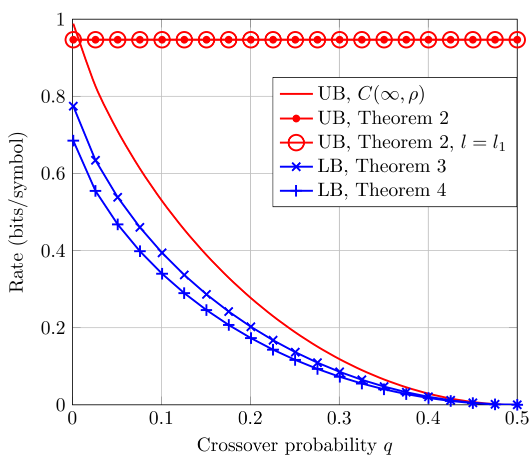

Fig. 6. Bounds on $C(\sigma, \rho)$ as a function of q for $(\sigma, \rho) = (0.78, 0.7)$.

with $\sigma = 0.7$ and $\sigma = 1.3$, respectively. Some of the bounds are stair functions because of the presence of floor and ceiling functions in their mathematical expressions. Bounds derived in Theorem 2 and 4 are depicted according to their numerical simulations, which may be conservative due to suboptimality of solutions of involved optimization problems.

In Fig. 4, the upper bound in Theorem 2 is zero for $\rho < 0.3$. Indeed, for such low energy arrival, it results $\sigma < 1 - \rho$ and $C(\sigma, \rho) = 0$ (cf. III-A2). Upper and lower bounds are shown to be very close for low ($\rho < 0.4$) and high ($\rho > 0.95$) energy arrival rates. The behavior of $C(\sigma, \rho)$ for low energy arrival rate is different when $\sigma > 1$. As shown in Fig. 5, it is possible to reliably communicate at nonzero rate for any $\rho > 0$.

In both Figs. 4 and 5, the tighter lower bounds are provided by either Theorem 3 or 4. The best upper bounds are provided by either Theorem 1 or 2. It is noteworthy that for $\rho > 0.5$, the upper bounds on $C(\sigma, \rho)$ in Theorem 2 when $l = l_1$ is coincident with that in Theorem 2 when optimizing over all l. This exhibits that for large ρ the Hamming weight constraint for length-l_1 subsequences becomes the most sensitive one among all Hamming weight constraints for subsequences. Fig. 6 shows capacity bounds as a function of the crossover probability q for fixed $(\sigma, \rho) = (0.78, 0.7)$. It is shown that lower bounds are close to the capacity of a system with infinite battery even for small battery size, i.e., $\sigma < 1$.

VI. Conclusion

We investigated the capacity of EHBSCs with (σ, ρ)-power constraint, where ρ is the energy arrival rate and σ is the battery size. Upper and lower bounds on the channel capacity were derived in terms of the normalized exponent of the cardinality of the set of feasible input sequences. Upper bounds were derived by relaxing some of the constraints posed by the harvesting process. Lower bounds were derived by applying the binary entropy-power inequality and by employing variations of the harvest-and-transmit scheme. Numerical results showed that proposed upper and lower bounds are close, providing effective evaluations of channel capacity and insights for practical signaling schemes useful in energy harvesting communications.

Acknowledgment

This work was supported in part by the SUTD-ZJU Research Collaboration under Grant SUTD-ZJU/RES/01/2014 and the MOE ARF Tier 2 under Grant MOE2015-T2-2-104.

References

[1] S. Sudevalayam and P. Kulkarni, "Energy harvesting sensor nodes: Survey and implications," *IEEE Commun. Surv. Tutor.*, vol. 13, no. 3, pp. 443–461, Third 2011.

[2] S. Ulukus, A. Yener, E. Erkip, O. Simeone, M. Zorzi, P. Grover, and K. Huang, "Energy harvesting wireless communications: A review of recent advances," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 3, pp. 360–381, Mar. 2015.

[3] D. Gunduz, K. Stamatiou, N. Michelusi, and M. Zorzi, "Designing intelligent energy harvesting communication systems," *IEEE Commun. Mag.*, vol. 52, no. 1, pp. 210–216, Jan. 2014.

[4] O. Ozel and S. Ulukus, "Achieving AWGN capacity under stochastic energy harvesting," *IEEE Trans. Inf. Theory*, vol. 58, no. 10, pp. 6471–6483, Oct. 2012.

[5] K. Tutuncuoglu, O. Ozel, A. Yener, and S. Ulukus, "Binary energy harvesting channel with finite energy storage," in *Proc. IEEE Int. Symp. Inf. Theory*, July 2013, pp. 1591–1595.

[6] ——, "Improved capacity bounds for the binary energy harvesting channel," in *Proc. IEEE Int. Symp. Inf. Theory*, June 2014, pp. 976–980.

[7] Y. Dong and A. Özgür, "Approximate capacity of energy harvesting communication with finite battery," in *Proc. IEEE Int. Symp. Inf. Theory*, June 2014, pp. 801–805.

[8] Y. Dong, F. Farnia, and A. Ozgur, "Near optimal energy control and approximate capacity of energy harvesting communication," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 3, pp. 540–557, Mar. 2015.

[9] O. Ozel, K. Tutuncuoglu, S. Ulukus, and A. Yener, "Capacity of the discrete memoryless energy harvesting channel with side information," in *Proc. IEEE Int. Symp. Inf. Theory*, June 2014, pp. 796–800.

[10] ——, "Capacity of the energy harvesting channel with energy arrival information at the receiver," in *Proc. IEEE Inf. Theory Workshop*, Nov. 2014, pp. 331–335.

[11] W. Mao and B. Hassibi, "On the capacity of a communication system with energy harvesting and a limited battery," in *Proc. IEEE Int. Symp. Inf. Theory*, July 2013, pp. 1789–1793.

[12] ——, "New capacity upper bounds and coding aspects for some channels with causal CSIT," in *Proc. IEEE Int. Symp. Inf. Theory*, June 2015, pp. 126–130.

[13] V. Jog and V. Anantharam, "A geometric analysis of the AWGN channel with a (σ, ρ)-power constraint," *IEEE Trans. Inf. Theory*, vol. 62, no. 8, pp. 4413–4438, Aug. 2016.

[14] S. Shamai and A. D. Wyner, "A binary analog to the entropy-power inequality," *IEEE Trans. Inf. Theory*, vol. 36, no. 6, pp. 1428–1430, Nov. 1990.