An Achievable Rate Region for Superposed Timing Channels

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***Abstract*—A multiple-access channel where point processes are randomly transformed by timing channels and then superposed is considered. An achievable rate region for the *K*-user channel is established. A single-user achievable rate in the presence of "many" interfering users is proposed. Results are applied to exponential server timing channels.**

I. Introduction

A. Background and Motivation

Queueing is an ubiquitous phenomenon in data networks [1] and represents the archetypal example of mechanism that may blur the timing information encoded into packet timings. From the information-theoretic perspective, the capacity of a queue was first investigated by Anantharam and Verdú in [2]. In that seminal paper, they found the capacity of the queue with exponential server, that has been also referred to as the exponential server timing channel (ESTC) in many subsequent works, proposed capacity bounds for the queue with general server, and laid the foundation for several future developments [3]–[8].

Almost all of the works that followed were devoted to the study of point-to-point timing channels as embodied in the single-server queue with one input arrival process and one output departure process. In the presence of multiple arrival processes, each linked to a different user, a multiuser communication through the queue occurs: this model was studied in [6], where the capacity region was found, and it was shown that it was achievable via time-division. From both a practical and theoretical perspective, it is of interest the case where the output of different queues is superposed and a receiver observes the superposition of departure processes in order to decode the messages.

In this paper, we address the problem of finding an achievable rate region for superposed point processes, where each point process in the superposition is the output of a timing channel, that displaces the input process points (see Fig. 1).

The model in this paper is similar to, but different from, the photon multiple-access channel with no dark current and peak-power constraint only [9]. The photon channel was widely adopted for the study of optical communications [10]–[13]. In the single-user photon channel, a Poisson counting process N_t is driven by a waveform $\lambda(t)$, that depends on the message to transmit, and a possibly nonzero constant intensity term λ_0, that is termed *dark current*. The intensity of the process N_t is thus $\lambda(t) + \lambda_0$, and the capacity is the maximal mutual information between $\lambda(t)$ and N_t. The single-user capacity of the photon channel was found by Kabanov in [10] and Davis in [11], and the error exponent by Wyner in [12], [13]. In the multiuser setting, K parallel channels, each driven by the intensity function of a different user, are summed. The counting process is, in this case, $N_t = N_t^0 + N_t^1 + \cdots + N_t^K$, where N_t^0 and N_t^k are the counting process generated by dark current and user k, respectively. The main result for the multiple-access channel was given by Lapidoth and Shamai in [9], where the capacity region of the two-user channel was found. The analogy with a timing channel is not straightforward. In the photon channel, each encoder generates an intensity function, rather than a set of points. However, following the approach of Wyner [12], a timing channel with memoryless insertions and deletions can be regarded as a the photon channel. The difference with queueing systems is therefore that packets are neither deleted nor inserted in queues with infinite buffer (provided that the system is stable), and that the channel has memory.

A different problem, that may share some non-trivial analogies with the present work, is that of finding the Poltyrev capacity (normalized logarithmic intensity) of point processes subject to random displacements. This problem was investigated by Anantharam and Baccelli for both the single-user and the multiuser AWGN channel in [14] and [15], respectively.

B. Contributions

In this paper we derive an achievable rate region for the multiple-access channel given by the superposition of point processes, each being the output of the timing channel of a different user. We provide the intuition behind our results, that are also specialized to the case where each timing channel is the ESTC (queue with exponential server).

Notations: The convex hull of the set $\mathcal{A}$ is denoted $\mathrm{co}(\mathcal{A})$, and $\bigwedge$ denotes the logical 'and' operator.

II. System Model

Two users share a common resource (time) and want to communicate towards a common receiver (see Fig. 1). From the mathematical standpoint, the signal transmitted by each user is a point process on the real line. Instead of describing the point process via a random measure, we represent each point process through the random sequence of its points.

Consider a number of channel uses per user equal to n, that is also the number of points transmitted by each user.

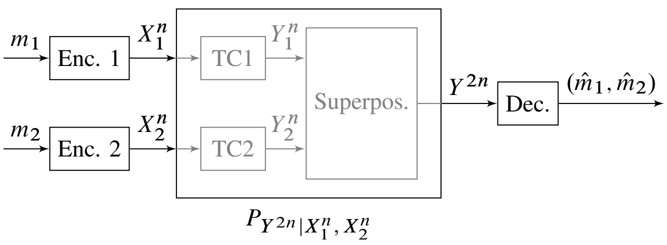


FIG. 1: System model.

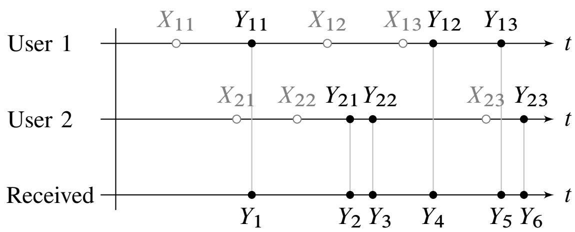


FIG. 2: Point process realization.

User i transmits the point sequence X_i^n, that is randomly transformed into Y_i^n by the timing channel (TC) $P_{Y_i^n|X_i^n}$. The superposition Y^{2n} of the two processes Y_1^n and Y_2^n is received (see Fig. 2). The overall superposed timing channel (STC) is formally described by the conditionally probability distribution $P_{Y^{2n}|X_1^n,X_2^n}$. Each TC $P_{Y_i^n|X_i^n}$ can have memory, and the superposition channel is dimension-mismatched.

We assume that the point processes of the two users are stationary with same intensity λ. Denote (R_1, R_2) a rate pair. The objective of this paper is to find an achievable rate region $\mathcal{R} \ni (R_1, R_2)$ of rate pairs valid for a large class of STCs. The extension of the system model to the K-user case is straightforward.

III. RATE REGION

From the general theory of multiple access channels, it is known that the following rate region $\mathcal{R}_n$ is achievable [16]:

$$\mathcal{R}_n\colon \quad \begin{aligned} R_1 &\leqslant \tfrac{1}{n} I(X_1^n; Y^{Kn}), \\ &\vdots \\ R_K &\leqslant \tfrac{1}{n} I(X_K^n; Y^{Kn}), \end{aligned} \tag{1}$$

and that the capacity region $\mathcal{C}$ is the closure of $\cup_{n\geqslant 1}\mathcal{R}_n$. In this section we will find a single-letter characterization of an achievable region for a large class of STCs on the basis of the multi-letter characterization (1) for the two-user channel. Considerations about the K-user channel are postponed in § IV.

A. Converse Region

As converse region we intend a region of forbidden rate pairs. Since each user in the two-user STC cannot have higher capacity than that achievable alone (without the presence of the other user), any rate $R_i \geqslant C_i$, where C_i is the capacity of the single-user TC, is not achievable by user i. Formally, denote M_i^{2n} the $2n$-binary vector such that $M_{ik} = 1$ if $Y_k \in Y_i^n$, and $M_{ik} = 0$ otherwise, *i.e.*, M_{ik} is a marking process that indicates whether point k in the superposition belongs to the point process of user i ($M_{ik} = 1$) or not ($M_{ik} = 0$). Since

$$I(X_i^n; Y^{2n}) \leqslant I(X_i^n; Y^{2n}, M_i^{2n}) = I(X_i^n; Y_i^n), \tag{2}$$

then any rate R_i higher than

$$C_i = \lim_{n\to\infty} \frac{1}{n} \sup_{P_{X_i^n}} I(X_i^n; Y_i^n) \tag{3}$$

is not achievable.

B. Two-User Achievable Rate Region

Theorem 1. Let define the two following regions: the time-division (TD) region $\mathcal{R}_{\text{TD}}$,

$$\mathcal{R}_{\text{TD}}\colon\ \frac{R_1}{C_1} + \frac{R_2}{C_2} \leqslant 1, \tag{4}$$

and the single-user region with penalty $\mathcal{R}_{\text{SU}}$,

$$\mathcal{R}_{\text{SU}}\colon\ \begin{aligned} R_1 &\leqslant C_1 - 2, \\ R_2 &\leqslant C_2 - 2, \end{aligned} \tag{5}$$

where all rates are expressed in bits per channel use. An achievable rate region for the two-user with same intensity STC is given by the convex hull (denoted co$(\cdot)$) of the union of the two regions $\mathcal{R}_{\text{TD}}$ and $\mathcal{R}_{\text{SU}}$:

$$\mathcal{R} = \text{co}(\mathcal{R}_{\text{TD}} \cup \mathcal{R}_{\text{SU}}). \tag{6}$$

Explicitly, region $\mathcal{R}$ is characterized as follows,

$$\mathcal{R}\colon \begin{cases} \frac{R_1}{C_1-2} + \frac{R_2}{2} \leqslant \frac{C_2}{2} \bigwedge \frac{R_1}{2} + \frac{R_2}{C_2-2} \leqslant \frac{C_1}{2} & \text{if } \frac{1}{C_1} + \frac{1}{C_2} \leqslant \frac{1}{2}, \\ \frac{R_1}{C_1} + \frac{R_2}{C_2} \leqslant 1 & \text{otherwise,} \end{cases} \tag{7}$$

where all rates are expressed in bits per channel use.

The achievable rate region $\mathcal{R}$ is shown in Figs. 3A-3B.

Proof: The proof is divided in three steps.

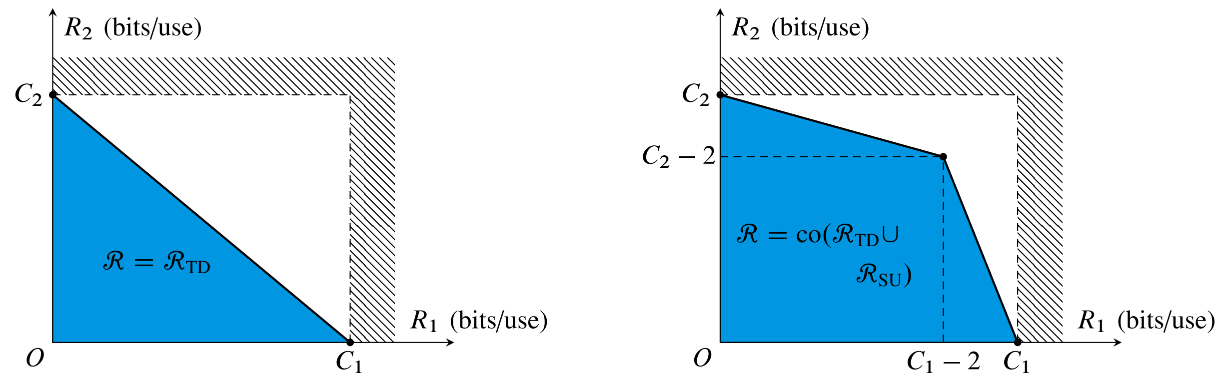
Step 1: Time-division. Since $(R_1, R_2) = (C_1, 0)$ and $(R_1, R_2) = (0, C_2)$ are two achievable pairs, all rates on the line between $(C_1, 0)$ and $(0, C_2)$ are achievable, therefore the region $\mathcal{R}_{\text{TD}}$ in (4) is achievable via TD.

Step 2: Single-User with Penalty. The two-user channel region $\mathcal{R}_n$ is given by

$$\mathcal{R}_n\colon\ \begin{aligned} R_1 &\leqslant \tfrac{1}{n} I(X_1^n; Y^{2n}), \\ R_2 &\leqslant \tfrac{1}{n} I(X_2^n; Y^{2n}). \end{aligned} \tag{8}$$

Applying the chain rule to the mutual information term in (8) for user 1 yields

$$I(X_1^n; Y^{2n}) = I(X_1^n; Y^{2n}, M_1^{2n}) - I(M_1^{2n}; X_1^n | Y^{2n}), \tag{9}$$



(A) $\frac{1}{C_1} + \frac{1}{C_2} > \frac{1}{2}$. (B) $\frac{1}{C_1} + \frac{1}{C_2} \leqslant \frac{1}{2}$

FIG. 3: Achievable rate regions.

where M_1^{2n} marks the points of user 1 in Y^{2n} (as formally defined in § III-A). By finding a lower bound of (9) we identify a region that is included in $\mathcal{C}$, hence it is achievable. The first term in (9) satisfies

$$I(X_1^n; Y^{2n}, M_1^{2n}) = I(X_1^n; Y_1^n), \tag{10}$$

since Y_1^n is a sufficient statitistic of $[Y^{2n}, M_1^{2n}]$ for X_1^n. The second term in (9) is bounded as

$$I(M_1^{2n}; X_1^n | Y^{2n}) = H(M_1^{2n}|Y^{2n}) - H(M_1^{2n}|Y^{2n}, X_1^n) \tag{11}$$
$$\leqslant H(M_1^{2n}), \tag{12}$$

where the inequality follows because entropy is nonnegative and conditioning cannot increase entropy. Denote $\mathcal{M}_1^{2n}$ the set of possible realizations of the marking process M_1^{2n}. Regarding each realization M_1^{2n} as a $2n$-binary string yields

$$H(M_1^{2n}) \leqslant \log |\mathcal{M}_1^{2n}| = \log \binom{2n}{n} \tag{13}$$

where $|\cdot|$ stands for the cardinality of the set in the argument, and the equality follows since the number of 1 in each $2n$-vector realization M_1^{2n} is equal to n. Using the fact that $\binom{n}{pn} = 2^{nH_2(p)+O(\ln n)}$, where $H_2(p) = -p \log_2 p - (1-p) \log_2(1-p)$, it results

$$\lim_{n\to\infty} \frac{1}{n} H(M_1^{2n}) \leqslant 2 \text{ bits/use}. \tag{14}$$

Another perhaps simpler way to derive the above bound is as follows:

$$H(M_1^{2n}) \leqslant 2nH(M_1) \leqslant 2nH_2(1/2) = 2n \text{ bits}. \tag{15}$$

Using (15) in (12), and (10) in (9) results in

$$\frac{1}{n} I(X_1^n; Y^{2n}) \geqslant \frac{1}{n} I(X_1^n; Y_1^n) - 2 \text{ bits/use}, \tag{16}$$

and taking the supremum over all feasible input distributions yields

$$R_1 \geqslant C_1 - 2 \text{ bits/use}. \tag{17}$$

A similar relation holds for user 2, that is,

$$R_2 \geqslant C_2 - 2 \text{ bits/use}. \tag{18}$$

Therefore, the rate region in (5) is achievable.

Step 3: Time-sharing. By using time-sharing [16], the achievable region $\mathcal{R}$ is given by the convex hull of the union of the two regions $\mathcal{R}_{\text{TD}}$ and $\mathcal{R}_{\text{SU}}$ (cf. (6)). The two achievable regions $\mathcal{R}_{\text{TD}}$ and $\mathcal{R}_{\text{SU}}$ overlap, as depicted in Figs. 4A-4B. In particular, it results $\mathcal{R}_{\text{SU}} \subset \mathcal{R}_{\text{TD}}$ whenever

$$(C_1 - 2, C_2 - 2) \in \mathcal{R}_{\text{TD}} \iff \frac{C_1 - 2}{C_1} + \frac{C_2 - 2}{C_2} \leqslant 1 \tag{19}$$
$$\iff \frac{1}{C_1} + \frac{1}{C_2} \geqslant \frac{1}{2}. \tag{20}$$

In this case, the achievable region $\mathcal{R}$ is given by time-division only, *i.e.*, $\mathcal{R} = \mathcal{R}_{\text{TD}}$, that is shown in Fig. 4A. Otherwise, $(C_1-2, C_2-2) \notin \mathcal{R}_{\text{TD}}$: in this case, using time-sharing allows to achieve all rates on the lines between $(0, C_2)$ and $(C_1 - 2, C_2 - 2)$, and between $(C_1 - 2, C_2 - 2)$ and $(C_1, 0)$, *i.e.*,

$$\mathcal{R} = \text{co}(\mathcal{R}_{\text{TD}} \cup \mathcal{R}_{\text{SU}}) = \big\{R_1 \geqslant 0, R_2 \geqslant 0 : \frac{R_1}{C_1-2} + \frac{R_2}{2} \leqslant \frac{C_2}{2} \bigwedge \frac{R_1}{2} + \frac{R_2}{C_2-2} \leqslant \frac{C_1}{2}\big\}, \tag{21}$$

as depicted in Fig. 4B. □

IV. K-USER ACHIEVABLE RATE REGION

Theorem 2. Let K be the number of users and define the two following regions: the time-division region $\mathcal{R}_{\text{TD}}$,

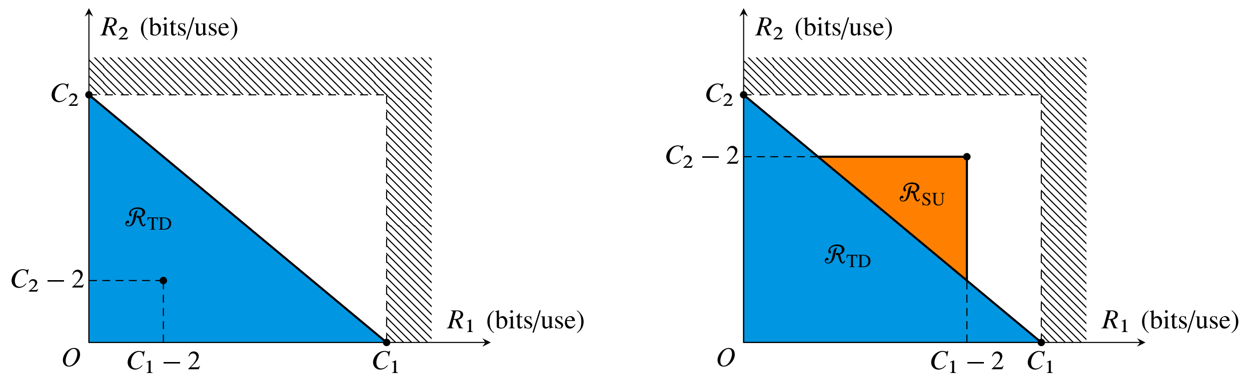
$$\mathcal{R}_{\text{TD}}\colon \sum_{i=1}^{K} \frac{R_i}{C_i} \leqslant 1, \tag{22}$$

and the single-user region with penalty $\mathcal{R}_{\text{SU}}$,

$$\mathcal{R}_{\text{SU}}\colon R_i \leqslant C_i - [K \log K - (K-1) \log(K-1)],\ i \in [1:K]. \tag{23}$$

A simpler yet weaker (but asymptotically equivalent) single-user region is:

$$\mathcal{R}'_{\text{SU}}\colon\ R_i \leqslant C_i - \log(Ke),\ i \in [1:K]. \tag{24}$$



(A) (B)

FIG. 4: Illustration of the regions in the proof of Theorem 1.

The two following rate region $\mathcal{R}$ and $\mathcal{R}'$, with $\mathcal{R}' \subset \mathcal{R}$, are achievable: $\mathcal{R} = \mathrm{co}(\mathcal{R}_{\rm TD} \cup \mathcal{R}_{\rm SU})$, $\mathcal{R}' = \mathrm{co}(\mathcal{R}_{\rm TD} \cup \mathcal{R}'_{\rm SU})$.

Proof: The time-division region $\mathcal{R}_{\rm TD}$ follows similarly to the proof of Theorem 1. Along similar lines, in order to derive the single-user region $\mathcal{R}_{\rm SU}$ we can focus on user 1 because the problem is symmetric. From the chain rule we have

$$I(X_1^n; Y^{Kn}) = I(X_1^n; Y^{Kn}, M_1^{Kn}) - I(M_1^{Kn}; X_1^n | Y^{Kn}), \quad (25)$$

where M_1^{Kn} marks the points of user 1 in Y^{Kn}. We find a lower bound of (25) as follows:

$$I(X_1^n; Y^{Kn}) \geqslant I(X_1^n; Y_1^n) - H(M_1^{Kn}). \quad (26)$$

An upper bound of the second term is given by

$$H(M_1^{Kn}) \leqslant \log \binom{Kn}{n} \quad (27)$$

since since the number of 1 in each Kn-vector realization M_1^{Kn} of the marking process is n. As $n \to \infty$ with K fixed one has

$$\lim_{n\to\infty} \frac{1}{n} \log \binom{Kn}{n} = K \log K - (K-1)\log(K-1). \quad (28)$$

Therefore, it results asymptotically

$$\lim_{n\to\infty} \frac{1}{n} H(M_1^{Kn}) \leqslant K \log K - (K-1)\log(K-1). \quad (29)$$

Another perhaps simpler way to derive the above bound is as follows:

$$\begin{aligned} \tfrac{1}{n} H(M_1^{Kn}) &\leqslant \tfrac{1}{n} Kn H(M_1) \\ &\leqslant K H_2(1/K) \\ &= K\left(\tfrac{1}{K}\log K - \left(1 - \tfrac{1}{K}\right)\log\left(1 - \tfrac{1}{K}\right)\right) \\ &= K \log K - (K-1)\log(K-1). \end{aligned} \quad (30)$$

Using (30) in (26) results in

$$\begin{aligned} \frac{1}{n} I(X_1^n; Y^{Kn}) \geqslant{}& \frac{1}{n} I(X_1^n; Y_1^n) \\ & - [K \log K - (K-1)\log(K-1)]. \end{aligned} \quad (31)$$

Taking the supremum over feasible input distributions yields

$$R_1 \geqslant C_1 - [K \log K - (K-1)\log(K-1)]. \quad (32)$$

Thus the rate interval specified in (23) is achievable. Since the penalty term (30) can be expanded as

$$K \log K - (K-1)\log(K-1) = \log(Ke) + O\left(\tfrac{1}{K}\right), \quad (33)$$

and, in particular, $K \log K - (K-1)\log(K-1) < \log(Ke)$, (33) is further lower bounded as follows,

$$R_1 \geqslant C_1 - \log(Ke), \quad (34)$$

that yields (24). □

Remark 1. Note that (23), for $K = 2$, reduces to $R_i \leqslant C_i - 2\log 2$, that yields the bounds in (5) when the base of the logarithm is equal to 2 and rates are thus expressed in bits per channel use (as in Theorem 1), while the bound in (24) is slightly weaker, $R_i \leqslant C_i - \log(2e)$. The penalty term increases from 2 bits/use to $\log_2(2e) \approx 2.44$ bits/use. □

V. APPLICATIONS

In this section we apply the above bounds to the superposition of ESTCs, *i.e.*, each TC $P_{Y_i^n|X_i^n}$ is given by the queue with exponential server. Each point in the process represents an absolute (cumulative) time and is referred to a single packet. Following [2], the message of user i is encoded into a sequence of interarrival times A_i^n, that are related to points X_i^n as follows: $X_{ik} = \sum_{l=1}^{k} A_{il}$, that is, X_{ik} is the arrival time of packet k in the queue of user i. Similarly, denoting D_i^n the packet interdeparture times from the queue of user i, it results $Y_{ik} = \sum_{l=1}^{k} D_{il}$, that is, Y_{ik} is the departure time of packet k from the queue of user i. Suppose that, for all users, arrival and service rates are λ and μ, respectively. Hence, the single-user capacity $C_i \equiv C$ (irrespective of i). In [2], it was found that the single-user capacity (with output rate λ) is given by $C = \log(\mu/\lambda)$ information units (depending on the base of the logarithm) per channel use. Rates in $\mathcal{R}_{\rm SU}$ (cf. (5)) assume the form

$$R_i \leqslant \log_2 \frac{\mu}{\lambda} - 2 = \log_2 \frac{\mu}{4\lambda} \text{ bits/use}. \quad (35)$$

The achievable region $\mathcal{R}$ is given by (cf. (7)):

$$\mathcal{R}\colon \begin{cases} \frac{R_1}{\log_2 \frac{\mu}{4\lambda}} + \frac{R_2}{2} \leqslant \frac{1}{2}\log_2 \frac{\mu}{\lambda} \bigwedge \\ \qquad \frac{R_1}{2} + \frac{R_2}{\log_2 \frac{\mu}{4\lambda}} \leqslant \frac{1}{2}\log_2 \frac{\mu}{\lambda} & \text{if } \mu \geqslant 16\lambda, \\ R_1 + R_2 \leqslant \log_2 \frac{\mu}{\lambda} & \text{otherwise,} \end{cases} \quad (36)$$

where all rates are expressed in bits per channel use. In particular, (36) shows that rates larger than those in $\mathcal{R}_{\text{TD}}$ are achievable when

$$\frac{1}{\mu} \leqslant \frac{1}{8}\cdot\frac{1}{2\lambda}. \quad (37)$$

That is, time-division is suboptimal when the average service time $1/\mu$, that is responsible for blurring the timing information, is less than a certain fraction (in this case equal to one-eighth) of the average time interval between two next packets $1/(2\lambda)$. When $\mu/\lambda \to \infty$, the penalty for each user (that is equal to at most 2 bits/use) becomes negligible with respect to C, since $C \to \infty$.

A similar result holds for the K-user channel. Let us present an asymptotic expression following the same spirit of the above derivation. The following region is achievable (cf. (23)-(24)):

$$R_i \leqslant \log\frac{\mu}{\lambda} - P,\ i \in [1\colon K], \quad (38)$$

where P is the penalty term equal to either $K\log K - (K-1)\log(K-1)$ or the more conservative $\log(Ke)$. It results $\mathcal{R} = \mathcal{R}_{\text{TD}}$ whenever

$$(C-P,\cdots,C-P) \in \mathcal{R}_{\text{TD}} \iff C \leqslant \tfrac{K}{K-1}P, \quad (39)$$

hence time-division is suboptimal when

$$\frac{\mu}{\lambda} \geqslant e^{\frac{K}{K-1}P} = Ke + O(\log K) \quad (40)$$

where P in (40) is expressed in nats. There exists a (positive, monotonically decreasing) sequence $(\epsilon_k)_{k\geqslant 2}$ such that $\mu/\lambda \geqslant K(e+\epsilon_K)$, and

$$\frac{1}{\mu} \leqslant \frac{1}{e+\epsilon_K}\cdot\frac{1}{K\lambda}. \quad (41)$$

We can interpret (41) in the same spirit of (37). The term $1/(K\lambda)$ is the average time interval between two next packets in the superposed point process. Therefore, as above, when the average service time is less than a certain fraction (in this case $1/(e+\epsilon_K) \to 1/e$ as $K \to \infty$) of the average time interval between two next points, there exists a code that allows to achieve rates beyond $\mathcal{R}_{\text{TD}}$.

VI. Discussion

We proposed an achievable rate region for the STC given by the convex hull of the union of the time-division region $\mathcal{R}_{\text{TD}}$ and the single-user region $\mathcal{R}_{\text{SU}}$. Let discuss the intuition behind the backoff from the single-user capacity in $\mathcal{R}_{\text{SU}}$. For clarity of exposition, consider $K = 2$. The backoff can be regarded as the (maximum) amount of information per channel use needed by the receiver to identify, among all received points, the subset of points of the user to decode. Since two points are received (on average) per channel use and each point may belong to either user 1 or user 2 with equal probability, it results that 2 bits/use are sufficient to discriminate the points between users. The 2-bits-per-use penalty is referred to as the *maximum* backoff since it is derived from the inequality $I(M_1^{2n}; X_1^n|Y^{2n}) \leqslant H(M_1^{2n})$ (cf. (12)). For a large class of TCs, time-division might result in $I(M_1^{2n}; X_1^n|Y^{2n}) \leqslant \epsilon$ with $\epsilon > 0$ (independent on n). The idea is that time-division guarantees the separation of the two users. Indeed, by appropriately assigning beforehand each user to a different subinterval, M_1^{2n} is approximately *a priori* known and $H(M_1^{2n})$ becomes negligible (the whole term $n^{-1}I(M_1^{2n}; X_1^n|Y^{2n})$ is vanishing in this case). At the other extreme, when Y_1^n and Y_2^n are two point processes that "often interleave," M_1^{2n} becomes an alternating binary sequence with the same number of occurrences of each binary symbol, and $H(M_1^{2n})$ may approach $2n$ bits. This might be the case for Poisson point processes: indeed, the fact that Poisson processes are closed under superposition and thinning suggest that $H(M_1^{2n}) \approx 2n$ bits (when both Y_1^n and Y_2^n have same intensity).

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